APPLICATION

FOR

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METHOD FOR VISUALIZING DATA

TECHNICAL FIELD

The present invention relates generally to
graphical computer systems and, more specifically, to a
5 method and system for visualizing data.

BACKGROUND OF THE INVENTION

Scientific visualization involves displaying large
sets of data generated in the process of investigating a
scientific or engineering problem. In many engineering
10 applications, large linear systems of equations arise. Such
applications include, for example, simulation of nonlinear
microwave circuits, robotic control, or power systems
design. Linear systems of equations are typically expressed
in the form

15 $$Ax = b, \qquad (1)$$

where $A \in R^{n \times n}$ or $A \in C^{n \times n}$, i.e., A is a square n-dimensional
matrix with real or complex elements, $x \in R^{n \times 1}$ and $b \in R^{n \times 1}$, or
$x \in C^{n \times 1}$ and $b \in C^{n \times 1}$, i.e., x and b are either real or complex
n-dimensional vectors. The entries of the matrix A are
20 values that arise from the physical system involved. For
example, the matrix A is often a Jacobian matrix related to
a nonlinear algebraic equation associated with a physical
system, e.g., a conductance matrix of an electronic circuit,
where x is a voltage vector and b is a current vector.

25 A matrices characterize both numerical and
structural properties of the physical system. Numerical or
quantitative properties are indicated by the elements of A

being, for example, "real," "complex," "positive," "negative," "integer," or "floating point" to name just a few. Structural or qualitative properties are related to the position or indexing of the elements of the matrix. The

5 position of an element in a matrix is indexed by integers indicating the row and column position of the element. Note that matrices are 2-dimensional arrays and some physical systems may involve higher dimensional arrays such as 3-dimensional arrays. In the case of a 3-dimensional array,

10 for example, the position of elements in the array are indexed by a row, a column, and a plane. Characterization of structural properties of a matrix include, for example: dense matrices, in which many of the elements of the matrix are non-zero; sparse matrices, in which many of the elements

15 of the matrix are zero; Hermitian or symmetric matrices, in which a matrix equals its adjoint; and skew-symmetric matrices, in which the elements of the transpose of the matrix have the opposite sign from the elements of the matrix.

20   Many numerical methods exist for the solution of systems of equations expressed in the form of equation (1); see, for example, Kendall E. Atkinson, "An Introduction to Numerical Analysis," (Wiley, 1989), and Yousef Saad, "Iterative Methods for Sparse Linear Systems," (PWS

25 Publishing Company, 1996). Numerical methods for the solution of these systems of equations include reordering, preconditioning, factoring, and substitution. The choice of numerical method for the solution of equation (1) is guided by the numerical and structural characteristics of the

30 physical systems modeled.

There are conventional software packages that allow a user to store and display data arrays, such as

matrices. For example, commercial software packages such as
MATLAB® by the Math Works Inc. or MATHEMATICA® by Wolfram
Research. These software packages, however, may not be able
to display a conductance matrix associated with the
5   operation of an electrical circuit with millions of
components. The main difficulty with conventional packages
is the limitation placed on allocation of memory provided
for the storage of data arrays, thus limiting the ability of
conventional packages to display large arrays. The
10   deficiencies of the conventional software packages show that
a need still exists for a method and system which permit
visualization of large sets of data.

SUMMARY OF THE INVENTION

The difficulty of choosing an appropriate and
15   efficient numerical method for the solution of the
engineering problem may be overcome by graphical
visualization of the data. In particular, it may be
insightful to graphically view a matrix such as the A
matrix of equation (1). For example, a microprocessor
20   integrated circuit (IC) may use four signal types: power
signals, clock (timing) signals, control signals, and data
signals. Each signal type is described by a reactive
network having one or more resistive, capacitive, and
inductive components. Such a network may contain 1-10
25   million components for each signal type. Each such network
is described by an equation of the form $Ax = b$. In
particular, the direct current (dc) solution of a
microprocessor (IC) power signal network may be represented
by an equation $Ax = b$, with A symmetric and positive
30   definite. In the case for which A is symmetric and
positive definite, Cholesky factorization and backward
substitution may be applied to solve the equation $Ax = b$.

To meet this and other needs, and in view of its purposes, the present invention provides a method and system for visualizing data arrays represented in the form of a plurality of data values. A grid is generated based on the plurality of data values. Each data value is associated with a geometric shape. The geometric shapes associated with the data values are placed on the grid. The visual and geometric information placed on the grid are reported to a user in graphical form.

In addition, it is possible to identify one of a plurality of numerical attributes associated with each data value of the plurality of data values, and associate with each numerical attribute one of a plurality of visual attributes. Then, each data value is associated with a geometric shape having one of the plurality of visual attributes. The geometric shapes associated with each data value of the plurality of data values are placed on the grid. The visual and geometric information placed on the grid are reported to a user in graphical form.

Alternatively, the present invention provides a method and system for visualizing data provided in the form of a geometric representation. A plurality of data values are extracted from the geometric representation. A graphic representation of the plurality of data values is generated. The graphic representation is displayed to a user.

It is to be understood that both the foregoing general description and the following detailed description are exemplary, but are not restrictive, of the invention.

BRIEF DESCRIPTION OF THE DRAWING

The invention is best understood from the following detailed description when read in connection with the accompanying drawing. Included in the drawing are the following figures.

Fig. 1 is a flow chart illustrating steps in a method for visualizing data arrays in accordance with an embodiment of the present invention;

Fig. 2 is a flow chart illustrating steps in a method for visualizing data arrays in accordance with an embodiment of the present invention;

Fig. 3 is a flow chart illustrating steps in a method for visualizing data provided in the form of a geometric representation;

Fig. 4 is an exemplary graphical representation of a simple dense array that is helpful in understanding an embodiment of the present invention;

Fig. 5 is an exemplary circuit schematic;

Fig. 6 is a block diagram representation of an exemplary power distribution circuit of an integrated circuit;

Fig. 7 is a more detailed circuit model for R_{IC_S} and R_{IC_G} of Fig. 6;

Fig. 8 shows an exemplary physical model for R_{IC_S} and R_{IC_G} of Fig. 7; and

Fig. 9 illustrates metalization levels and inter-level connections of Fig. 8 as sparsely connected resistive grids.

DETAILED DESCRIPTION OF THE INVENTION

5   In integrated circuit physical design systems, for example, special shapes associated with the integrated physical circuit layout are described by a syntax of a language such as, for example, Graphics Language 1 (GL1) or Graphic Design Language (GDS). These languages provide
10   mechanisms to describe three-dimensional space, orientation, and parametric information such as power service terminal (PST), logic service terminal, and IC package to IC interconnections. The shapes described by languages such as GL1, GDS, or the like are placed on a grid that represents
15   the physical area of an integrated circuit (a chip). The geometric description language may then be used to build a set of photolithographic masks. Each mask represents a different step in the manufacture of an integrated circuit. Different geometric characteristics of the lithographic
20   masks represent transistors, wires, insulation, or the like, of the integrated circuit. The integrated circuit may be manufactured on a silicon substrate with insulated interconnects (wires). The geometric shapes involved in an integrated circuit are typically on a microscopic scale.
25   Such a manufacturing process is a planar, or layered, process in which transistors may occupy the lower levels and the interconnects and insulation occupy the remaining higher levels.

       A physical design system such as, for example, the
30   Cadence™ system may graphically display or print out information, such as a grid including geometric shapes,

coded in a geometric description language such as GL1. Physical design software systems typically have large memory capacity to accommodate the shapes involved in the design of an integrated circuit. A physical design system may accommodate designs having millions of shapes, such as the shapes involved in an integrated circuit which are typically on a microscopic scale. Hence, a physical design system may graphically illustrate data arrays with millions of data variables.

Fig. 1 is a flow chart illustrating steps in a method for visualizing data arrays in accordance with an embodiment of the present invention. In step **102**, a multi-dimensional grid is generated. The position of data values in an array, for example, the rows, columns, and planes, may be used to generate the grid. In other words, the grid may be treated as a template for a numerical array, or the grid may act as a reference in a similar manner to a coordinate system. The grid may subsequently be used as a reference for the placement of geometric shapes.

In step **104**, arbitrary geometric shapes are associated with the data values in the array. That is, each entry or data value is identified with a geometric shape such as, for example, a line, a circle, a rectangle, a triangle, or a dodecahedron. Numerical properties of the data values such as, for example, magnitude and sign as well as the type of data values, for example, real or complex may be used to determine the choice of geometric shape. Numerical properties may be used to determine the sort of geometric shape identified with a data value as well as the relative size and appearance of the shape. For example, a positive, real number may be represented by a solid square;

a negative, real number may be represented by an open
square. Thus, a graphical representation of the dense array

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

may be constructed as illustrated in Fig. 4.

5   For example, consider a system that uses graphics
capabilities, such as an integrated circuit physical design
system. In such a physical design system, geometric shapes
may be used to represent circuit components including
transistors, interconnects (wires), and insulation. Such
10   circuit components are represented by data values detailing
various technical specifications of the components. For
example, the dimensions of a transistor length or width may
be specified in micrometers (e.g., 0.6×10^{-6} meters) and the
electrical resistance of a conductor may be specified in
15   ohms (e.g., $1.5 \times 10^{-3} \Omega$).

          In step **106**, the geometric shapes associated with
the data values are placed on the grid. The grid may be
multi-dimensional so as to correspond to a multi-dimensional
array. The positioning of a geometric shape on the grid is
20   related to the index of a data value entry of the array.

          In step **108**, the information contained in the
grid, including the geometric shapes, is reported to a user.
It is preferable to report the information to a user on a
computer display or in the form of a printout.

The present invention may take advantage of the capability of a physical design software system to graphically display numerical data arrays. As detailed in the steps shown in Fig. 1, numerical and structural

5 properties of data arrays may be represented in a form that is recognized by a physical design system. The data array displayed, for example, in a printout makes it possible to visualize properties of a data array.

If a data array is sparse, for example, then by

10 visualizing the data array it may be possible to observe certain patterns. Sparsity patterns may lead to the choosing of suitable numerical methods, or reordering schemes with which to treat the data array. Sparsity patterns may also provide insight into the possibility of

15 parallel processing of the data array. An integrated circuit visualization of the array may be used to determine circuit properties such as, for example, points of high current or points of low voltage. If the data values in the data array represent numerical data that is part of a larger

20 computational process, then visualizing the data array may help to detect computationally intensive portions of the process.

Fig. 2 is a flow chart illustrating steps in a method for visualizing data arrays in accordance with an

25 embodiment of the present invention. In the same way as in step **102** of Fig. 1, a grid is generated in step **202**. In step **204**, numerical attributes associated with data values of the data array are identified. Numerical attributes associated with data values may include the sign of a data

30 value and the range of magnitude of a data value. In step **206**, visual attributes are associated with numerical attributes identified in step **204**. Visual attributes may

include, for example, color, hue, brightness, and
saturation. Visual attributes may be used to highlight
numerical properties of the data values of the data array.
For example, the range and distribution of the numerical
5   magnitude of data values may be highlighted by a variety of
colors and changes in brightness.

In step **208**, geometric shapes having the proper
visual attributes are associated with the data values in the
array. In other words, each data value, having a numerical
10 attribute, is associated with a geometric shape, having a
visual attribute, through the association performed in step
206. For example, a geometric shape associated with a data
value with a magnitude belonging to a particular range of
magnitudes has the color associated with that range of
15 magnitudes.

In step **210**, similarly to step **106** of Fig. 1, the
geometric shapes associated with the data values are placed
on the grid. In step **212**, which is the same as step **108** of
Fig. 1, the information contained in the grid including the
20 geometric shapes is reported to a user. It is preferable to
report the information to a user on a computer display or in
the form of a printout.

In the foregoing, a method is described for
visualizing data arrays provided in the form of data values.
25 The data are associated with geometric shapes placed on a
grid. It is also possible, in accordance with another
embodiment of the present invention, to visualize data
provided in the form of a geometric representation. Fig. 3
is a flow chart illustrating steps in a method for
30 visualizing data provided in the form of a geometric
representation. A graphic representation may be coded, for

example, in a geometric description language such as GL1, GDS, or the like. In step **302**, data values, such as numerical data values, are extracted from a geometric representation. Consider the case in which the geometric representation is an integrated circuit layout. In this case, numerical data values are obtained by extracting power buses and network resistance from the integrated circuit layout. The numerical data values thus obtained may then be organized in a data array such as a matrix.

In step **304**, a graphic representation of the data values is generated. Such a graphic representation may be, for example, a graphic representation of a sparse matrix. If the graphic representation is an integrated circuit layout, such a sparse matrix may be a conductance matrix obtained, for example, as part of quiescent drain current analysis. A graphic representation of a sparse matrix may be, for example, a grid corresponding to rows and columns of a matrix in which a colored dot is placed on the grid for each non-zero entry in the matrix.

In step **306**, the graphic representation is displayed to a user. A physical design software system such as the Cadence™ system may be used to display a graphic representation, such as a representation of a sparse matrix. A physical design software system may be used to print a colored graphic representation of a sparse matrix.

The use of a physical design software system for the implementation of the present invention is advantageous because such systems can handle large volumes of data. It may in fact be the case that, for some applications, the amount of memory required to store a graphic representation, coded in a geometric description language such as GL1, is

larger than the amount of memory required to store the
extracted data values.

EXAMPLE

The following example is included to more clearly
demonstrate the overall nature of the invention: a method
for visualization of numerical matrices. The entries of the
matrices may include multi-dimensional, linear, and/or non-
linear numerical expressions. This example is exemplary,
not restrictive, of the invention.

Matrices are useful constructs both in theoretical
and applied mathematical analysis. Specifically, in the
field of dynamical systems matrices are useful in the course
of both "steady-state" (time independent) and "dynamic"
(time dependent) analysis. Both steady-state and dynamic
analyses may be performed in relation to dynamic systems
arising in the fields of electrical, hydraulic, thermal, and
mechanical engineering.

Kirchoff's Current Law (KCL) is a specific example
from circuit analysis. KCL is a conservative law which
states that the sum of the currents "entering" and "exiting"
a node in an electrical network equals zero. A node is
defined as the junction of two or more electrical
components. Fig. 5 is an exemplary circuit schematic
illustrating five such nodes, labeled 0 through 4,
connecting between resistors labeled R0, R1, R2, R3, R4, R5,
R6, and R7. Table 1 shows a "net list" associated with the
circuit of Fig. 5. The "net list" is a tabular
representation of the circuit or electrical network of Fig.
5. Table 1 provides all of the information included in the
circuit diagram of Fig. 5.

TABLE 1
"Net List"

MODEL NAME	FROM NODE	TO NODE	VALUE
R0	0	1	1
R1	0	3	2
R2	0	2	3
R3	1	2	4
R4	3	2	5
R5	1	4	6
R6	2	4	7
R7	3	4	8
I_s	0	4	I

5

Another equivalent representation of the circuit
of Fig. 5 may be obtained by applying KCL. In matrix form
the representation of the circuit of Fig. 5 is denoted by
the following equation:

$$G*v = \begin{bmatrix} 6 & -1 & -3 & -2 & 0 \\ -1 & 11 & -4 & 0 & -6 \\ -3 & -4 & 19 & -5 & -7 \\ -2 & 0 & -5 & 15 & -8 \\ 0 & -6 & -7 & -8 & 21 \end{bmatrix} * \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = i = \begin{bmatrix} I_s \\ 0 \\ 0 \\ 0 \\ -I_s \end{bmatrix},$$

10

where G is a matrix that represents the conductance of
resistive components R0 through R7 in the circuit, v is a
vector of node voltages, and i is a vector of branch
currents. Thus, the dimension, or "size", of G, v, and i
15 each depends on the number of nodes in the circuit. In
other words, the greater the number of nodes, the larger the
"size" of matrix G and vectors v and i.

A circuit that results in a relatively large
conductance matrix G is the power distribution circuit of an

integrated circuit (IC). Fig. 6 is a block diagram representation of such a circuit. The circuit of Fig. 6 includes: an external power supply with an "earth" reference, a source voltage, and a ground voltage; a

5 resistor R_{PKG}, representing the resistance of the IC package; a resistor R_{IC}, representing the resistance of the IC power distribution metalization layers; and IC transistors. Fig. 6 illustrates that the voltage delivered to the IC transistors is "divided" by R_{PKG} and R_{IC}, where V_{ps} is lowered

10 by the voltage drop across R_{PKG_S} and R_{IC_S}, as given by the equation:

$$V_{PFET_S} = V_{ps_S} - V_{PKG_S} - V_{IC_S} ,$$

and V_{ps_G} is raised by the voltage drop across R_{PKG_G} and R_{IC_G}, as given by the equation:

15 $$V_{NFET_G} = V_{ps_G} + V_{PKG_G} + V_{IC_G} .$$

Fig. 7 shows a more detailed circuit model for R_{IC_S} and R_{IC_G}, where each "level" of IC metalization is modeled as a separate resistor. A contemporary IC may have ten levels to connect V_{ps_S} to V_{PFET_S}, and an additional ten levels to

20 connect V_{NFET_G} to V_{ps_G}.

Fig. 8 shows the corresponding physical model for R_{IC_S} and R_{IC_G} of Fig. 7, where a metalization level is shown as a rectangle, and an inter-level connection or "via" is represented as a square.

25 Fig. 9 illustrates the levels and vias of Fig. 8 as sparsely connected resistive grids. A contemporary IC may have approximately five million resistors in each resistive block. Therefore, the resulting conductance

matrix G may be of on the order of 3.5 million X 3.5
million. Such a matrix is very large.

As applied to this exemplary embodiment, the
present invention provides a way to organize the rows and
columns of conductance matrix G. Matrix G may be organized
so that the entries of matrix G correspond to the physical
position of nodes in the circuit. The physical position of
nodes in the circuit is shown, for example, in Fig. 9. The
following matrix-vector equation G*v=i corresponds to such a
reorganization of matrix G for the circuit diagrams of Figs.
7 and 8.

$$
\begin{bmatrix}
1 & -1 \\
-1 & 2 & -1 \\
& -1 & 2 & -1 \\
& & -1 & 2 & -1 \\
& & & -1 & 2 & -1 \\
& & & & -1 & 2 & -1 \\
& & & & & -1 & 2 & -1 \\
& & & & & & -1 & 2 & -1 \\
& & & & & & & -1 & 2 & -1 \\
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& & & & & & & & & & & & & & -1 & 2 & -1 \\
& & & & & & & & & & & & & & & -1 & 2 & -1 \\
& & & & & & & & & & & & & & & & -1 & 2 & -1 \\
& & & & & & & & & & & & & & & & & -1 & 2 & -1 \\
& & & & & & & & & & & & & & & & & & -1 & 1 \\
\end{bmatrix}
\cdot
\begin{bmatrix}
v_{PKG_S} \\ v_{LV_S} \\ v_{LM_S} \\ v_{V4_S} \\ v_{M4_S} \\ v_{V3_S} \\ v_{M3_S} \\ v_{V2_S} \\ v_{M2_S} \\ v_{V1_S} \\ v_{M1_S} \\ v_{M1_G} \\ v_{V1_G} \\ v_{M2_G} \\ v_{V2_G} \\ v_{M3_G} \\ v_{V3_G} \\ v_{M4_G} \\ v_{V4_G} \\ v_{LM_G} \\ v_{LV_G} \\ v_{PKG_G}
\end{bmatrix}
=
\begin{bmatrix}
i_{PKG_S} \\ i_{LV_S} \\ i_{LM_S} \\ i_{V4_S} \\ i_{M4_S} \\ i_{V3_S} \\ i_{M3_S} \\ i_{V2_S} \\ i_{M2_S} \\ i_{V1_S} \\ i_{M1_S} \\ i_{M1_G} \\ i_{V1_G} \\ i_{M2_G} \\ i_{V2_G} \\ i_{M3_G} \\ i_{V3_G} \\ i_{M4_G} \\ i_{V4_G} \\ i_{LM_G} \\ i_{LV_G} \\ i_{PKG_G}
\end{bmatrix}
$$

where all zero entries have been left blank, all resistive
values have been set to one ohm, and the corresponding
resistors are connected serially. Hence, conductance matrix

G is organized such that adjacent resistors in the circuit appear as adjacent entries in matrix G.

Although illustrated and described herein with reference to certain specific embodiments, the present invention is nevertheless not intended to be limited to the details shown. Rather, various modifications may be made in the details within the scope and range of equivalents of the claims and without departing from the spirit of the invention. The present invention can further be embodied in the form of computer-implemented processes and apparatus for practicing such processes, for example, and can be embodied in the form of computer program code embodied in tangible media, such as floppy diskettes, fixed (hard) drives, CD ROM's, magnetic tape, fixed/integrated circuit devices, or any other computer-readable storage medium, such that when the computer program code is loaded into and executed by a computer, the computer becomes an apparatus for practicing the invention.